Dual Nature of Matter

LIST OF FORMULAE

1. Energy of a photon $E = h\upsilon = \frac{hc}{\lambda}$
2. Number of photon emitted per second $N = \frac{P}{E}$
3. Momentum of photon $P = mc = \frac{h\upsilon}{c} = \frac{h}{\lambda} = \frac{E}{c}$
4. Equivalent mass of photon $m = \frac{h\upsilon}{c2} = \frac{E}{c2} = \frac{h}{c\lambda}$
5. Work function $W_0 = h\upsilon_0 = \frac{hc}{\lambda 0} =$
6. Kinetic energy of photoelectron is given by Einstein's photoelectric equation:
   $K_{max} = \frac{1}{2}mV^2 = h\upsilon - W_0 = h(\upsilon - \upsilon_0) = h\left(\frac{c}{\lambda} - \frac{c}{\lambda 0}\right)$
7. If V_0 is the stopping potential, the maximum kinetic energy of the ejected photoelectron,
   $K = \frac{1}{2}mv_{max}^2 = eV_0$
8. Kinetic energy of De-Broglie Waves $K = \frac{1}{2}mv2 = P^2/2m$
9. Momentum of De-Broglie Waves $P = \sqrt{2mK}$
10. Wavelength of De-Broglie Waves $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{(2mK)}}$
11. De –Broglie Wavelength of an electron beam accelerated through a potential difference of V volts is
    $\lambda = \frac{h}{\sqrt{(2meV)}} = \frac{1.23}{\sqrt{V}}$ nm $= \frac{12.27}{\sqrt{V}}$ A^0
12. De –Broglie Wavelength associated with gas molecules of mass m at temperature T kelvin is
    $\lambda = \frac{h}{\sqrt{(2mKT)}}$ K = Boltzmann constant
13. The value of hc = 12400eV A^0
14. The Value of $\frac{hc}{e} = 1240 \times 10^{-9}$ eV m

Level-I:- Numerical direct formula based (1 mark,2 mark)

1. If the maximum kinetic energy of electrons emitted by a photocell is 4ev.What is the Stopping potential?
2. What is the energy associated in joules with a photon of wavelength 4000A^0?
3. The photoelectric cut-off voltage in a certain experiment is 1.5V.Waht is the maximum Kinetic energy?
4. Calculate the work function of a metal in eV.If its threshold wavelength is 6800A^0?
5. What is the momentum of a photon of energy 120 MeV?
6. What is the de-broglie wavelength (in A^0) associated with an electron accelerated through a Potential of 100V?

7.Calculate the ratio of de-Broglie wavelength associated with a deuteron moving with velocity 2V and an alpha particle moving with velocity V?

8. The work function of cesium metal is 2.14eV.When light of frequency 6 X 10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the (a) Maximum kinetic energy of the emitted electron and (b) Stopping potential of the emitted photoelectron?

9.In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency

of incident light is found to be 4.12 X 10^{-15}Vs.Calculate the value of Planck's constant.

10. The threshold frequency for a certain metal is 3.3 X10^{14}Hz is incident on the metal;   Predicts the cut-off voltage for photoelectric emission.

LEVEL-2

1.An electron and an alpha particle have same kinetic energy. Which of these particles has the shortest de-Broglie wavelength?
2.The de Broglie wavelength of an electron is 1 A^0. Find the velocity of the electron.
3.Determine the accelerating potential required for an electron to have a de-Broglie wavelength of 1 Å
4.An electron, an alpha particle and a proton have the same kinetic energy, which one of these particles has (i) the shortest and (ii) the largest, de, Broglie wavelength?
5.In an experiment on photo electric emission , following observations were made;
(i) wave length of incident light = 1.98 x 10^{-7}m
(ii) stopping potential = 2.5 V.
Find (a) kinetic energy of photo electrons with maximum speed
(b) work function & (c) threshold frequency
6. Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

(a) Find the energy and momentum of each photon in the light beam,

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and

(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

7. In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant.

8. The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cutoff voltage for the photoelectric emission.

9. The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

10. Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

LEVEL -3

LEVEL-III:- 10 numericals challenging/difficulty level (1 mark, 2 marks, 3marks)

1. A radio transmitter operates at a frequency of 880 KHz and a power of 1KW.Find the

Number of photons emitted per second.

2 A blue lamp mainly emits light of wavelength 4500 A^0. The lamp is rated at 150 W and 8% of the energy is emitted as visible light. How many photons are emitted by the lamp per second?

3 Calculate the de-broglie wavelength of a proton of a momentum 2.55 X 10^{-22} $Kgms^{-1}$?

4 The work function of Cesium is 2.14 eV.Find (a) the threshold frequency for Cesium, and (b) the wavelength of the incident light if the photocurrent is brought to zero by a stopping potential of 0.60eV?

5 If the photoelectrons are to be emitted from a potassium surface with a speed of 6 X 10^6 ms^{-1}.What frequency of radiation must be used? (Threshold frequency for potassium is 4.22 X 10^{14} Hz, h= 6.6 X10^{-34} Js and m_e = 9.1 X 10^{-31} kg)

6. A sheet of silver is illuminated by monochromatic ultraviolet light of wavelength = 1810 A^0. What is the maximum energy of the emitted electron? Threshold wavelength of of silver is 2640 A^0.

7. By how much would be stopping potential for a given photosensitive surface go up if the frequency of the incident radiation were to be increased from 4 X10^{15} Hz to 8 X 10^{15}? Given h= 6.6 X10^{-34} Js, e = 1.6 X 10^{-19} C and c = 3 X 10^8 ms^{-1}?

8. The photosensitive threshold wavelength for a metal is 10000 A^0.When light of wavelength 5461 A^0 is incident on it, the retarding potential in Millikan's experiment is 1.02 Calculate the value of Planck's constant?

9. When light of wavelength 400 nm is incident on the cathode of a photocell, the stopping recorded is 6V.If the wave of the incident light is increased to 600nm.Calculate the new stopping potential?

10 The two identical photocathodes receive light of frequencies f_1 and f_2.If the velocity of the photoelectron (of mass m) coming out are respectively v_1 and v_2 , then show that $v_1^2 - v_2^2 = \frac{2h}{m}$ (f_1-f_2).

ANSWER

LEVEL -1

1. 4 volt
2. $E = \frac{hc}{\lambda}$ =4.96 X 10 $^{-19}$joule
3. 1.5 eV
4. W =1.825 eV
5. 64 X 10 $^{-24}$ Kg m/sec
6. 1.23 A^0
7. $\frac{\lambda d}{\lambda_\alpha} = \frac{h/p_d}{h/p_\alpha} = \frac{m_\alpha v_\alpha}{m_d v_d} = \frac{1}{1}$
8. (a) K_{max} = 0.34 eV (b) 0.34 V
9. h = slope X e = 6.59 X 10^{-34} js
10. eV0 = h(υ - $υ_0$) , V0 = $\frac{h(υ - υ0)}{e}$ = 1.37V

LEVEL-2

1.: Alpha particle
2. 7.3 x 10 ^{6} m/s
3. V = 150.6 V
4.

$$\lambda \frac{h}{\sqrt{2mE_k}} \ \alpha \ \frac{1}{\sqrt{m}}$$

5. (a) K_{max} = 2.5eV (b) work function = 3.76eV (c) threshold frequency = 9.1x 10^{14}Hz

6. Wavelength of the monochromatic light, λ = 632.8 nm = 632.8 × 10^{-9} m

Power emitted by the laser, P = 9.42 mW = 9.42 × 10^{-3} W

Planck's constant, h = 6.626 × 10^{-34}Js,Speed of light, c = 3 × 10^8 m/s

Mass of a hydrogen atom, m = 1.66 × 10^{-27} kg

(a) The energy of each photon is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{632.8 \times 10^{-9}} = 3.141 \times 10^{-19} \text{ J}$$

The momentum of each photon is given as:

$$P = \frac{h}{\lambda}$$

$$= \frac{6.626 \times 10^{-34}}{632.8} = 1.047 \times 10^{-27} \text{ kg m s}^{-1}$$

(b) Number of photons arriving per second, at a target irradiated by the beam = *n*

Assume that the beam has a uniform cross-section that is less than the target area.

Hence, the equation for power can be written as:

$$P = nE$$

$$\therefore n = \frac{P}{E}$$

$$= \frac{9.42 \times 10^{-3}}{3.141 \times 10^{-19}} \approx 3 \times 10^{16} \text{ photon/s}$$

(c) Momentum of the hydrogen atom is the same as the momentum of the photon, $p = 1.047 \times 10^{-27} \text{ kg m s}^{-1}$

Momentum is given as:

$p = mv$ Where, v = Speed of the hydrogen atom

$$\therefore v = \frac{p}{m}$$

$$= \frac{1.047 \times 10^{-27}}{1.66 \times 10^{-27}} = 0.621 \text{ m/s}$$

7. The slope of the cut-off voltage (*V*) versus frequency (□) of an incident light is given as:

$$\frac{V}{\nu} = 4.12 \times 10^{-15}\ \text{Vs}$$

V is related to frequency by the equation:

$$h\nu = eV$$

Where, e = Charge on an electron = 1.6×10^{-19} C

h = Planck's constant

$$\therefore h = e \times \frac{V}{\nu}$$

$$= 1.6 \times 10^{-19} \times 4.12 \times 10^{-15} = 6.592 \times 10^{-34}\ \text{Js}$$

Therefore, the value of Planck's constant is 6.592×10^{-34} Js.

8. Threshold frequency of the metal, $\nu_0 = 3.3 \times 10^{14}$ Hz

Frequency of light incident on the metal, $\nu = 8.2 \times 10^{14}$ Hz

Charge on an electron, $e = 1.6 \times 10^{-19}$ C ,Planck's constant, $h = 6.626 \times 10^{-34}$Js

Cut-off voltage for the photoelectric emission from the metal = V_0

The equation for the cut-off energy is given as:

$$eV_0 = h(\nu - \nu_0)$$

$$V_0 = \frac{h(\nu - \nu_0)}{e}$$

$$= \frac{6.626 \times 10^{-34} \times (8.2 \times 10^{14} - 3.3 \times 10^{14})}{1.6 \times 10^{-19}} = 2.0292\ \text{V}$$

Therefore, the cut-off voltage for the photoelectric emission is 2.0292 V.

9. No , Work function of the metal, $\phi_0 = 4.2$ eV

Charge on an electron, $e = 1.6 \times 10^{-19}$ C ,Planck's constant, $h = 6.626 \times 10^{-34}$Js

Wavelength of the incident radiation, λ = 330 nm = 330×10^{-9} m

Speed of light, $c = 3 \times 10^8$ m/s ,The energy of the incident photon is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}} = 6.0 \times 10^{-19} \text{ J}$$

$$= \frac{6.0 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.76 \text{ eV}$$

It can be observed that the energy of the incident radiation is less than the work function of the metal. Hence, no

photoelectric emission will take place.

10. Wavelength of light produced by the argon laser, $\lambda = 488$ nm $= 488 \times 10^{-9}$ m

Stopping potential of the photoelectrons, $V_0 = 0.38$ V , 1eV $= 1.6 \times 10^{-19}$ J

$$\square\ V_0 = \frac{0.38}{1.6 \times 10^{-19}} \text{ eV}$$

Planck's constant, $h = 6.6 \times 10^{-34}$Js,Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Speed of light, $c = 3 \times 10$ m/s ,From Einstein's photoelectric effect, we have the relation involving the work function Φ_0 of the material of the

emitter as:

$$eV_0 = \frac{hc}{\lambda} - \phi_0$$

$$\phi_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 488 \times 10^{-9}} - \frac{1.6 \times 10^{-19} \times 0.38}{1.6 \times 10^{-19}}$$

$$= 2.54 - 0.38 = 2.16 \text{ eV}$$

Therefore, the material with which the emitter is made has the work function of 2.16 eV.

LEVEL-3

1. N = $\frac{p}{E}$ = $\frac{p}{h\upsilon}$ = 1.72 X10^{31} Photon/second
2. N = $\frac{8\%\ of\ P}{E}$ = $\frac{8P\lambda}{100hc}$ = 2.71 X10^{29} photon/second
3. λ = h/p = 0.026 A^0
4. (a)υ_0 = w/ h = 5.16 X 10^{14} Hz

   (b) λ = $\frac{hc}{eV0 + W}$ = 453.7 nm

5. K.E = $\frac{1}{2}$ mv^2 = $h(\upsilon - \upsilon_0)$

   υ = = $\frac{1}{2}$ mv^2/h + υ_0 = 6.7 X 10^{14} Hz

6. K_{max} = hc $(\frac{1}{\lambda} - \frac{1}{\lambda'})$ = 2.16 eV

7. $V_{02} - v_{01}$ = $\frac{h(\upsilon_{2} - \upsilon_{1}}{e}$ = 16 volt
8. eV0 = hc $(\frac{1}{\lambda} - \frac{1}{\lambda'})$

   h = 6.554 X 10^{-34} js

9. eV0 = hc (1/λ-w')

   Δ V0 = $V_{02} - v_{01}$ = $\frac{hc}{e}$ (1/λ-1/λ^')

   V_{02} = v_{01} -1.03= 6-1.03= 4.97 V
10. $\frac{1}{2}$ mv^2 = hf – hf_0 or v^2 = $\frac{2hf}{m}$ - $\frac{2hf0}{m}$

    $v_1^2 = \frac{2hf_1}{m} - \frac{2hf0}{m}$

    $v_2^2 = \frac{2hf_2}{m} - \frac{2hf0}{m}$

    $v_1^2 - v_2^2 = \frac{2h}{m}$ $(f_1 - f_2)$.